Modeling and Controller Design Based on the Max-Plus Algebra

by

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Abstract

This thesis studies modeling and controller design for discrete event systems based on the max-plus algebra. The max-plus algebra is an algebraic system in which the max operation is addition and the plus operation is multiplication. It has been widely studied and is regarded as one effective approach for the modeling, analysis and controller design of discrete event systems. The system representation by state-space equations, which is called the max-plus-linear system, can describe the event sequence of systems and the required times between adjacent events. Since the form of the state-space equations is similar to ones in modern control theory, various kinds of research developments in modern control theory have been applied to the max-plus algebra; for instance, internal model control, supervisory control, model predictive control and adaptive control, etc. Recently, related studies on these topics have been frequently reported. A main aim of this thesis is to make them applicable to the analysis and the control for large-scale problems or complicated systems. Specifically, related studies and theories on model predictive control are applied and extended.

First, chapter 1 describes the context of this thesis and introduces related studies, and chapter 2 gives mathematical preliminaries of the max-plus algebra. In chapter 3, the basic properties of the greatest subsolution that have not been clarified in past papers are inspected and are extended in the latter part of the chapter. The greatest subsolution is a solution method for linear equations in the max-plus algebra, and forms part of a key algorithm in designing controllers. We derive a relevancy between the greatest subsolution and the solution of a linear programming problem and obtain necessary and sufficient conditions for the existence of the strict solution, allowing performance evaluation to be accomplished more efficiently. Next, in chapter 4, we extend past research developments on model predictive control, and the idea of an inverse system is introduced. Since the system parameters had been handled as constants in past papers, the application scope to practical systems was limited. Hence, in this thesis, representing the system matrices in the form of linear-parameter-varying structures allows the system parameters to be represented by adjustable variables, and realizes a low cost system operation by online parameter adjustment. This study is further extended in chapter 5, where the system parameters are handled as variables that depend on the event counter. Thus, this could be applied to scheduling problems of production systems for assembly lines or batch processing lines that produce different kinds of products using the

same equipment.

Furthermore, chapter 6 introduces the idea of a selection parameter by which the application field of max-plus linear systems is extended to systems with selective lanes. In practice, we can find plenty of systems in which multiple machines or lanes for services are installed in parallel, in order to shorten the processing or waiting times. Utilizing the idea proposed in chapter 6, an optimal set of paths for un-processed parts or their input times for systems with selective lanes can be obtained by taking into account practical constraints such as delays for due dates, processing costs, and inventory costs.

Finally, in chapter 7, we design a new algorithm so that developments to model predictive control can be applied to large-scale problems or complicated systems. The series of procedures for controller design such as systems modeling, derivation of the state-space equations, and determination of optimal control inputs were performed manually in past papers, which was very time-consuming. Moreover, they had to be redone even when analyzing similar systems or when changing the prediction step number. In this thesis, a newly developed algorithm automates these procedures by specifying the precedence constraints, and the positions of inputs and outputs with three constraint matrices. This algorithm facilitates the analysis of large-scale problems or of systems with complicated precedence constraints. It is also useful for the design of a robust controller with a large prediction step number.

Contents

Chapter 1

Introduction

The primary focus of this thesis is on system modeling and controller design for DESs (Discrete Event Systems) based on the max-plus algebra. The max-plus algebra is an algebraic system in which addition and multiplication are defined by the max and + operators, respectively. It is utilized for modeling, performance evaluation, and controller design for DESs, and many related studies and developments have been reported recently.

1.1 Discrete Event Systems

Research developments in modern control theory have been applied to many areas of the process industry. They contribute to speedups, laborsaving, and cost reduction for various kinds of systems at present. Essential to controller design is the formulation of the states or the constraints of a system and the derivation of a system model for the controller. In order to construct a desirable controller, a suitable model of the system should be generated prior to anything else.

There are several kinds of states in practical systems: quantitative states such as temperature, angle, ON/OFF state of switches, etc., and qualitative states that are sometimes dependent upon the evaluators such as fast/slow, light/dark, etc. Furthermore, quantitative states are classified into two types: continuous state-values and discrete state-values. The former are physical values that can change infinitesimally and hence take all values in a specified range. On the other hand, the latter are values which at any time are restricted to one of a set of discrete states.

Systems in which the states change continuously are called Continuous Event Systems, while those in which the states change discretely are called DESs [6,18,46]. Systems in which there are both continuous and discrete quantities are called hybrid systems [1,7]. This thesis mainly focuses on modeling for DESs, however, modeling for hybrid systems can also be performed easily using the max-plus algebra.

In a broad sense, a DES is defined as any system in which the states change discretely. DESs can be categorized into several types based on the properties of the objective systems. One typical type of DES is a system on which the following constraints are imposed:

- Multiple events are synchronized, and subsequent events are invoked just after all previous events are finished
- After the invocation of an event, it takes a period of time before the next event is invoked

There are several representation methods for a system with the constraints described above; one is based on a TEG (Timed Event Graph) [22], and another is based on the max-plus algebra [23]. The TEG is a certain subclass of TPN (Timed Petri-Net) [60] whose properties are described as follows; each token has specific residence times or required times for the transition in all places, and all places have a single transition upstream and a single one downstream.

With the help of the TEG, we can describe event sequences for systems with precedence constraints, and lead times between consecutive events can also be formulated. Utilizing the max-plus algebra, DESs whose behaviors are expressed with the TEG can be represented by linear equations which have an independent variable called the event counter. Such a system representation is called a MPL (Max-Plus Linear) system.

The max-plus algebra is also called the max algebra or the $(\max, +)$ algebra. It is an algebraic system in which the max operation is addition and the + operation is multiplication, and it has attractive features that are familiar in conventional $(+, \times)$ algebra; commutativity and the distributive law hold, and properties of the linearity can be utilized in matrix calculations. Linear equations in the MPL system are represented in a form similar to the state-space equation in modern control theory, and research developments in the theory have recently been applied to the max-plus algebra. Therefore, system modeling based on the max-plus algebra is very closely involved with the TEG, which is one of several remarkable properties.

In this thesis, we focus on the analysis and the controller design for systems whose behavior is expressed in the form of the MPL system representation. Scheduling problems for production systems are used as illustrative examples, however, there remain several other DESs that are represented by the TEG and are referred to in the next subsection.

1.2 Related Studies and Reports

The basic idea of the max-plus algebra is proposed in Ref. [17], and is studied in the field of OR during the initial phase [30]. In the 80's, Ref. [21] revealed that the behavior of a certain class of production systems could be described using linear equations in the max-plus algebra, and also indicated that they could be used for effective performance evaluation of the systems. Subsequently, system modeling for DESs based on the max-plus algebra was frequently studied in Europe. Moreover, a book that covered the research results by the beginning of 90's was published [2]. Although the main topic in most studies by the early 90's was the performance evaluation for systems, there has been a gradual shift to the controller design for DESs since then.

Hereafter, let us review and summarize the related research papers from the following viewpoints:

- Analogical extensions from modern control theory
- Applications to practical systems
- Controller design methods

First, we summarize analogical extensions based on modern control theory. As described earlier, the state-space representation of MPL systems is similar to one in modern control theory. In conventional algebra, the state-space equations have 'time' as an independent variable for the representation of time-driven systems. On the other hand, MPL systems have the event counter as an independent variable for the representation of event-driven systems, which is analogous to the use of 'time' in conventional algebra. Since both equations bear a resemblance, it is expected that theories similar to those in conventional algebra could be developed in the max-plus algebra. For example, methods in modern control theory such as IMC (Internal Model Control) [33, 41], Supervisory control [44, 45, 65], MPC (Model Predictive Control) [16, 19, 24, 43], and Adaptive Control [39, 42] have been applied to the max-plus algebra.

In Refs. [20, 44, 63], the idea of Supervisory Control is applied to the max-plus algebra. For instance, Ref. [20] takes the idea of supervisory control for the TEG into account in the max-plus algebra; if the specification for a system is given by a set of firing times for the transitions, the control specification can be accomplished by delaying the firing times of controllable transitions. This is caused by control signals from the supervisor.

The idea of IMC, which is often utilized in controller design for chemical plants, is applied to MPL systems in Refs. [8, 38]. With the help of this study, a controller for adjusting the completion times to the desired ones is installed in a production system. A general result of this study is that the control inputs for perturbed systems can be made robust. Moreover, in Ref. [62], a controller based on IMC is developed and is tolerant to any disturbances permitted by a newly developed filter for reducing external disturbances.

In Refs. [9, 53, 54, 56], MPC is applied to MPL systems. MPC determines the control inputs by solving an optimization problem in which the performance of the system for a finite step is formulated [35, 61]. In Refs. [47, 51, 52], the ELCP (Extended Linear Complementarity Problem) is utilized for obtaining an optimal solution for the systems. The research developments in this paper are extended to systems in which white noise or modeling errors are imposed [10–13].

In addition to MPC, a theory of adaptive control is applied to the max-plus algebra. Ref. [14] realizes online control by combining a method for system identification and MPC, which is called adaptive MPC. This controller can adjust the states online even when the properties of a system are changed unexpectedly.

We can also find other research reports on the controller design for hybrid systems [31] and the parameter estimation problem of the state-space equations [5, 50, 57]. We expect these to be improved and further developed.

Next, we introduce several application fields for practical systems.

For the first example, a scheduling algorithm for assembly production systems [53] is described. A typical model of these systems is as follows; there are multiple machines installed, each machine processes parts in a specified time, and the manufactured parts are sent to the next machine or the next stage. Such a system is also frequently used for illustrative examples in this thesis [25,26,37,38]. The goal is to determine the input times such that the processing would complete at the desired time. In this model, the input times for the un-processed parts and the completion times for the manufactured parts correspond to the input variables and the output variables for the system, respectively. The starting times for the processing and the processing times are represented by the internal states and the system parameters, respectively. The constraint of such systems is that no processing can start until the corresponding machine is ready and all of the required parts have been inputted.

As similar examples, diagnosis and fault detection for batch-processing lines are introduced [34, 48, 49]. In such systems, the input times correspond to the starting times for the injection of a substance or solvent, and the output times are equal to the completion times for the outflow of the resulting substance. The system parameters are equal to the reaction times, which include the injection and the outflow times. The internal states are the starting times of the injection or the completion times of the outflow. The constraint of such systems is that each injection can be started only when the corresponding tank is empty and when all the required substance or solvent is ready. Faults in the system can be detected by evaluating errors of the internal states if deviations from the predicted values are greater than the thresholds.

Several problems for transportation planning in traffic networks utilizing the max-plus algebra are reported [15,32,40,64]. These problems can be formulated by setting the system variables in the following way: For instance, in railway networks, the inputs and the outputs correspond to the departure time from the railhead, and the arrival time at the terminus, respectively. The system parameters are equivalent to the travel times between the stations, and the internal states correspond to the departure or arrival times at the intermediate stations. The constraint that passengers can transfer to another train in an intermediate station should be satisfied. However, we can permit a train to depart even before all connecting trains have arrived. This constraint, that allows reversal of the event sequence, is called a soft constraint, while the conventional ones are referred to as hard constraints. Research on soft and hard synchronization can be found in Refs. [55, 58, 59].

In addition to the studies described above, we can find a report on the problem of flow-control of TCP in the field of communication networks [4]. Therefore, we expect that the application fields for the max-plus algebra will continue to expand.

For the last issue, we summarize the controller design methods for MPL systems. Unlike in conventional $(+, \times)$ algebra, an inverse element for addition is not defined in the max-plus algebra, which leads to the absence of an inverse matrix for matrix calculations. Hence, ingenuity is required for treating the state-space equations in MPL systems, and several solution methods have been developed and reported. Roughly speaking, there are two leading methods for solving linear equations in MPC. One of the methods is the ELCP, and the other is based on the greatest subsolution [8].

The solution method based on the ELCP was proposed in Ref. [47], and it has been frequently improved by the same authors. The ELCP determines the optimal control inputs by solving an optimization problem in which the constraints are given by the output prediction equation in MPC, and the objective function is formulated by taking the inputs and outputs of the system into account. While this method has the advantage that the objective function can be formulated flexibly, its calculation volume grows exponentially in proportion to the size of the problem [47, 51]. Hence, Ref. [53] proposes that the optimization problem could be solved by relaxing the equality conditions of the output prediction equations to inequalities.

On the other hand, the greatest subsolution is another solution method for equations in the max-plus algebra, and is a component of the Residuation Theory [2, 22, 23]. The greatest subsolution gives a solution of a linear equation, and methods based on the greatest subsolution determine control inputs by solving linear inequalities that are obtained by transforming the output prediction equation using constraints of the practical systems. Although these methods are inferior to the ELCP with respect to flexibility of the objective functions, they have the attractive advantage that the solutions are obtained by simple algebraic operations, which include ‘min’ and ‘-‘ in conventional algebra. Thus, they are useful in solving large-scale problems.

In this thesis, we adopt the greatest subsolution method for the controller design of MPL systems.

As described so far, remarkable progress in system modeling and controller design based on the max-plus algebra has recently been made. Nevertheless, most studies and reports are still under investigation, and require improvements for general use in practical systems. Moreover, even several fundamental properties of linear equations in the max-plus algebra remain unknown.

Hence, taking the related research results into account, this thesis directs them to become applicable to practical systems by extending them or developing new algorithms. Moreover, we perform basic analyses that have not been inspected in past papers.

An organization of this thesis is described in the following subsection.

1.3 Organization of This Thesis

As described in the previous subsection, this thesis examines system modeling in the max-plus algebra, especially focusing on controller designs in MPC. The main concerns in this thesis are as follows:

- System modeling of production systems based on the max-plus algebra, and the relevance with system representation utilizing the TPN
- Applications to practical systems: when the system parameters are adjustable or predetermined, and when there are selective parameters
- Fundamental properties of the greatest subsolution that play an important role in the controller design for MPL systems

The organization of each chapter is described next:

Chapter 2 gives mathematical preliminaries of the max-plus algebra. In addition to the definition of the basic operators, other operators that are required for the controller design are defined. A definition for the order of vectors and the operators for matrix calculations are also defined in this chapter.

Chapter 3 inspects the properties of the greatest subsolution that play an important role in designing controllers for MPL systems [28]. The greatest subsolution is given as a relaxed solution of the linear equation, and has the following properties:

- When there exists a strict solution, the greatest subsolution gives the maximum solution.
- When a strict solution does not exist, the greatest subsolution gives a best-approximated solution.

Namely, the greatest subsolution gives a unified and reasonable solution for the linear equation in the max-plus algebra in both cases above. Therefore, it is the basis of a key algorithm for deriving a control law in the field of controller design. However, there still remain several issues to be discussed regarding the properties of the greatest subsolution. In chapter 3, we focus on the following fundamental properties:

1. Formulation as an optimization problem
2. Uniqueness of the greatest subsolution

3. Necessary and sufficient conditions for the correspondence of the greatest subsolution with the strict solution

With respect to the first issue, we show that the greatest subsolution coincides with the optimal solution of a linear programming problem. This indicates that the flexibility of a controller design based on the greatest subsolution could be improved.

For the second issue, uniqueness of the greatest subsolution is discussed. A vector set in the max-plus algebra is a semi-ordered set, which implies an order relation between two vectors can be defined only if all the respective elements are in the same order relation. Therefore, there may exist more than one greatest subsolution. Chapter 3 shows the uniqueness of the greatest subsolution by proving that any vectors with which the greatest subsolution does not have an order relation cannot be greatest subsolutions.

For the third issue, we derive a necessary and sufficient condition for the correspondence of the greatest subsolution with the strict solution. This property has not been discussed thus far although there are two cases in which the greatest subsolution gives either the strict solution or the best-approximated solution. This result could facilitate the performance evaluation of the controller based on the greatest subsolution. Moreover, the necessary and sufficient condition for the uniqueness of the strict solution is also given.

Chapter 4 extends the basic theory for MPC introduced in Ref. [53]. The control laws of MPC for MPL systems are energetically studied and reported by the same authors [9, 53, 54, 56]. Since these studies utilize the ELCP method, the constraints for the inputs and the outputs are flexibly formulated. However, as for the system parameters, they have been handled as constants, and these are referred to as MPL systems with invariant system parameters. In designing a controller for production systems with the help of the max-plus algebra, the system parameters correspond to the processing abilities of internal machines. Moreover, in transportation planning of traffic or railway networks, the system parameters correspond to required times between stations. Since they are frequently adjusted in order to conform the output or arrival times to the desired ones, they should be handled as variables or adjustable parameters rather than fixed ones.

Therefore, in chapter 4, we introduce a MPL representation for systems with LPV (Linear-Parameter-Varying) structure, and derive inverse systems for them. An inverse system determines the control inputs for the system such that the control outputs follow the desired reference signals, which is a basic requirement in the field of controller design. Hence, it is essential to develop an inverse system for MPL systems with LPV structure. However, to the authors' knowledge, such accomplishments have not yet been reported. Moreover, derivations of the output prediction equation and the greatest subsolution have so far been performed assuming that the system parameters are constants [53].

Hence, we firstly give a MPL system representation for systems with LPV structure. Then, after deriving an output prediction equation, we prove that the equation can be also expressed in the form of LPV. In addition, the greatest subsolution is extended so that it can be applied to systems with LPV structure. Furthermore, we formulate an optimization problem for the inverse systems with LPV structure. Specifically, it is shown that both the constraints on the system parameters and the objective function are represented by linear functions of the parameters. As a consequence, the problem for determining optimized parameters can be reduced to a linear programming problem. This implies that the solution of MPL systems with LPV structure can be easily calculated.

In chapter 5, we further improve the idea of inverse system that is introduced in the previous chapter, and derive a control law of MPC for MPL system with adjustable system parameters.

As described earlier, past studies related to MPC have been performed under the constraints that system parameters remain constant. However, in practical systems, changing or adjusting the system parameters is common and sometimes inevitable. For example, the production system of an assembly line or a batch processing line produces multiple kinds of products on the same equipment, and the processing times for each kind of product differ. This means that the system parameters (processing times) are dependent upon the event counter (production no.). Furthermore, the number of workers may be adjusted according to the volume of the order. In such cases, the processing times will differ depending on the number of workers, and hence the system parameters are adjustable.

Therefore, the approach proposed in chapter 4 is useful for expanding the application field of MPL systems. However, the inverse system in chapter 4 has cases in which the system parameters vary abruptly. This is because the control inputs are determined under restrictions that the corresponding outputs should be equal to or less than the desired reference signal. Moreover, the system parameters are considered to be constant and not explicitly adjustable. In chapter 5, we propose a MPC design method in MPL systems with adjustable parameters by expanding the method in chapter 4. Using MPC, moderate changes of the system parameters can be accomplished by anticipating future events, and ensuring the outputs are within the desired reference signal. Moreover, a control method can be obtained at a lower cost. As the system parameters are defined to depend upon the event counter, their adjustment can be performed freely.

Hence, in chapter 5, we utilize the calculation method of the greatest subsolution derived in chapter 4 after we show that the prediction equation can be expressed as a linear summation function of the adjustable parameters.

Chapter 6 extends the application field of MPL systems by introducing the idea of selective parameters [29]. Utilizing the selective parameters, we extend the domain in which MPL systems can be applied, and develop a controller design method for systems with selective lanes. The specific interests of this chapter are described as follows:

1. Formulation of systems in which multple selective lanes are installed

2. Optimal solution taking into account the processing cost, the inventory cost, and the delay for the due dates

3. Effect of the prediction horizon and the receding horizon

The first issue is to apply MPL to systems in which multiple selective lanes are available. As described earlier, MPL systems are a subclass of Petri net and can describe the behavior of the TEG. While this subclass can describe systems whose transitions fire simultaneously, it cannot describe ones whose transitions fire selectively. However, in practice, there are plenty of systems in which multiple machines or lanes for services are installed in parallel for shortening the processing or waiting times. Production systems such as these cannot be modeled by a TEG.

Hence, by introducing the idea of selective parameters, we extend the domain in which MPL systems can be applied, and develop a controller design method for such systems. The parameters are functions of the event counter, and are assigned to each output transition. In [3], the algebraic formulation for free-choice Petri nets is discussed. However, due to the focus on modeling for free-choice nets in general, the advantages of the linear properties in the max-plus algebra tend to be lost. Therefore, chapter 6 deals with a certain class of TPN in which selector and joint places are incorporated with SISO TEG subnets. An MPL system with selective parameters is proposed for modeling this class of TPN.

The second issue is to determine an optimal set of selective parameters and corresponding control inputs. The control inputs are calculated utilizing the greatest subsolution [8, 22], and the optimal set is determined by evaluating an objective function for all feasible cases and adopting the minimum. Hence, the components that consist the objective function are significant. In many practical systems including production systems, their schedules are arranged in order that the completion times are within the due dates. However, other constraints such as the processing costs or the inventory costs are sometimes more critical than the due dates. Therefore, chapter 6 improves the objective function to determine an optimal control input taking into account practical constraints simultaneously. Specifically, the three components, the processing cost, the inventory cost, and the delay for the due dates, are included. To the authors' knowledge, the objective function taking into account all these components has not been reported so far.

The third issue is to examine the effect of the prediction horizon and the receding horizon in MPC. In chapter 5, it is shown that the prediction horizon should be set to a large number in order to obtain a robust control input, whereas the calculation time inflates steeply. Hence, we examine whether or not it is applicable to systems in which selective lanes are installed in parallel. Furthermore, we inspect the relation between the receding horizon and the performance of the solution.

Chapter 7 derives a new framework for the controller design and the analysis for MPL systems. With the help of the newly developed framework, the principal tasks for the controller design of MPC for MPL system such as the formulation of state-space equations, derivation of output prediction equations, and determination of optimal control inputs based on the greatest subsolution can be performed efficiently. As we described earlier, the studies on MPL systems are still under investigation, and require improvement for general use in practical systems. For example, in production systems, precedence constraints are frequently changed and the state-space equations should be restructured accordingly. Currently, the equations have to be derived manually, which is a very inefficient process. Therefore, we propose a new algorithm for deriving a state-space equation and determining an optimal control input with only a few manual procedures.

Generally, a controller design utilizing MPC is constructed from the following steps:

1. Model a control objective
2. Examine the constraints and derive a state-space equation
3. Derive an output prediction equation and determine a control law

The procedures described above would be laborsome under certain conditions. For instance, the derivation process of the state-space equations in step 2 is very time consuming for complicated systems. Even when an analogous system is to be analyzed, the design process must be started from step 1. When a robust controller should be designed, the prediction number must be large, and the prediction equation becomes complicated.

Hence, we propose a new algorithm to determine a system representation and an optimal control input. Utilizing the new algorithm, the state-space equation can be derived from only three constraint matrices and two parameter vectors. Accordingly, complicated systems can be analyzed easily.

Chapter 8 gives concluding remarks of this thesis, and lists the possible directions in future works.